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90. Proposed by J. MASCUS BOORMAN, Consultative Mechanician and Counselor at Law, Woodmere, Long Island, N. Y.

$$\begin{aligned}\text{Solve} \quad x^2 + y^2 + m(x+y) &= m^2 \dots\dots (1). \\ x^2 + y^2 + xy &= m^2 \dots\dots (2).\end{aligned}$$

I. Solution by COOPER D. SCHMITT, A. M., Professor of Mathematics, University of Tennessee, Knoxville, Tenn.; A. H. BELL, Hillsboro, Ill.; HON. JOSIAH H. DRUMMOND, LL. D., Portland, Me.; CHARLES C. GROSS, Libertytown, Md.; ELMER SCHUYLER, High Bridge, N. J.; and M. A. GRUBER, A. M., Washington, D. C.

$$\text{Subtracting we have } m(x+y) = xy \dots\dots (3).$$

$$\text{Squaring (3) } m^2(x^2 + 2xy + y^2) = x^2y^2.$$

$$\text{Multiplying (2) by } m^2, m^2(x^2 + xy + y^2) = m^4.$$

$$\text{Whence } x^2y^2 - m^2xy = m^4 \dots\dots (4),$$

$$\text{Solving as a quadratic, } xy = \frac{m^2}{2}(1 \pm \sqrt{5}) \dots\dots (5).$$

$$\text{Adding (5) and (2), } x^2 + 2xy + y^2 = \frac{m^2}{4}(6 \pm 2\sqrt{5}).$$

$$\text{Whence } x+y = \pm \frac{m}{2}(\sqrt{5} \pm 1).$$

$$\text{Similarly, } x-y = \pm \frac{m}{2}(\sqrt{5} - 2 \mp 6\sqrt{5}).$$

$$\text{Whence } x = \pm \frac{m}{4}(\sqrt{5} \pm 1 + \sqrt{5} - 2 \mp 6\sqrt{5}),$$

$$y = \pm \frac{m}{4}(\sqrt{5} \pm 1 - \sqrt{5} - 2 \mp 6\sqrt{5}),$$

II. Solution by M. A. GRUBER, A. M., War Department, Washington, D. C.

$$(1)-(2) \text{ gives } m(x+y) = xy. \quad \text{Whence } x+y = xy/m.$$

$$\text{Put } x = vy. \quad \text{Then } vy+y = vy^2/m, \text{ and } v+1 = vy/m.$$

$$\text{Whence } y = m(v+1)/v, \text{ and } x = vy = m(v+1).$$

Substituting the values of x and y in (1) or (2), we have

$$m^2(v+1)^2 + \frac{m^2(v+1)^2}{v^2} + \frac{m^2(v+1)^2}{v} = m^2.$$

Dividing by m^2 , clearing of fractions, expanding, etc., we obtain

$$v^4 + 3v^3 + 4v^2 + 3v + 1 = v^2.$$

Adding and subtracting $v^3 + 2v^2 + v$, we find

$$(v+1)^4 - v(v+1)^2 = v^2.$$

Completing square and extracting square root, we get

$$2(v+1)^2 - v = \pm v\sqrt{5}.$$

Expanding, transposing, and uniting terms, we obtain

$$2v^2 + (3 \mp \sqrt{5})v = -2.$$

Completing square, extracting square root, etc., we find

$$v = \frac{1}{2}[-3 \pm \sqrt{5} \pm \sqrt{-2(1 \pm 3\sqrt{5})}].$$

$$\text{Whence } v+1 = \frac{1}{2}[1 \pm \sqrt{5} \pm \sqrt{-2(1 \pm 3\sqrt{5})}],$$

$$\text{and } (v+1)/v = \frac{1}{2}[1 \pm \sqrt{5} \mp \sqrt{-2(1 \pm 3\sqrt{5})}].$$

$$\therefore x = m(v+1) \text{ and } y = m[(v+1)/v].$$

III. Solution by H. C. WHITAKER, A. M., Ph. D., Professor of Mathematics, Manual Training School, Philadelphia, Pa., and ALOIS F. KOVARIK, Instructor in Mathematics and Physics, Decorah Institute, Decorah, Iowa.

$$\text{Let } x = v + z, \text{ and } y = v - z. \quad 2v^2 + 2z^2 + 2mv = m^2. \quad 3v^2 + z^2 = m^2.$$

$$\text{Whence } v = \frac{1}{2}(1 \pm \sqrt{5})m, \text{ and } z = \pm \frac{1}{2}(-2 \mp 6\sqrt{5})m.$$

The four values of x and y are

$$x = \frac{1}{2}[1 + \sqrt{5} + \sqrt{(-2 - 6\sqrt{5})}]m; \quad y = \frac{1}{2}[1 + \sqrt{5} - \sqrt{(-2 - 6\sqrt{5})}]m.$$

$$x = \frac{1}{2}[1 + \sqrt{5} - \sqrt{(-2 - 6\sqrt{5})}]m; \quad y = \frac{1}{2}[1 + \sqrt{5} + \sqrt{(-2 - 6\sqrt{5})}]m.$$

$$x = \frac{1}{2}[1 - \sqrt{5} + \sqrt{(-2 + 6\sqrt{5})}]m; \quad y = \frac{1}{2}[1 - \sqrt{5} - \sqrt{(-2 + 6\sqrt{5})}]m.$$

$$x = \frac{1}{2}[1 - \sqrt{5} - \sqrt{(-2 + 6\sqrt{5})}]m; \quad y = \frac{1}{2}[1 - \sqrt{5} + \sqrt{(-2 + 6\sqrt{5})}]m.$$

IV. Solution by G. B. M. ZERE, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa., and J. SCHEFFER, A. M., Hagerstown, Md.

$$\text{Let } x + y = t, \quad xy = r.$$

$$\text{Then (1) and (2) become } t^2 - 2r + mt = m^2 \dots\dots (3), \quad t^2 - r = m^2 \dots\dots (4).$$

$$\text{Subtracting (4) from (3) we get } r = mt \dots\dots (5), \text{ or } t = r/m \dots\dots (6).$$

$$(5) \text{ in (3) gives } t^2 - mt = m^2. \quad \therefore t = \frac{1}{2}m(1 \pm \sqrt{5}).$$

$$(6) \text{ in (4) gives } r^2 - m^2 r = m^4. \quad \therefore r = \frac{1}{2}m^2(1 \pm \sqrt{5}).$$

Since $x + y = t$ and $xy = r$, we have

$$x = \frac{1}{2}[t \pm \sqrt{(t^2 - 4r)}] = \frac{1}{2}m\{1 \pm \sqrt{5} \pm \sqrt{[-2(1 \pm 3\sqrt{5})]}\}.$$

$$y = \frac{1}{2}[t \mp \sqrt{(t^2 - 4r)}] = \frac{1}{2}m\{1 \pm \sqrt{5} \mp \sqrt{[-2(1 \pm 3\sqrt{5})]}\}.$$

V. Solution by the PROPOSEE.

$x/m, y/m$; with eight roots (four each) [or sixteen by $\sqrt{m^2 = \pm m}$]; $x:y =$ ratio $X:Y$ with values $x = X$ times any- or every-thing; $y = Y$ any- or every-thing; $\therefore x/X = y/Y = m$ of an infinite degree and roots, all values from $\pm(1/\infty^n)$ to 0 to $\pm\infty^\infty$. And that m is a blind factor in $x = Xm$; $y = Ym$; — SOLVE CIRODE'S curious PROBLEM. (I) — (II) $\dots\dots m(x+y) = xy \dots\dots$ (III). Un-factor $x = Xm$; $y = Ym \dots\dots$ (A). $\therefore X^2 m^2 + Y^2 m^2 + m(Xm + Ym) = m^2 \dots\dots$ (I₁); $m(Xm + Ym) = XmYm \dots\dots$ (III₁) divided by m^2 are $X^2 + Y^2 + x + y = 1 \dots\dots$ (I_a);

$X + Y = XY \dots (III_a)$. Twice $(III_a) + (I_a)$ gives $(X + Y)^2 - (X + Y) = 1 \dots (IV)$. $\therefore X + Y = \frac{1}{2}(1 \pm \sqrt{5}) \dots (V)$. \therefore By (III_a) $XY = \frac{1}{2}(1 \pm \sqrt{5}) \dots (VI)$! $\therefore (V)(VI)$, (by *nature*) the constants of $X^2 - \frac{1}{2}(1 \pm \sqrt{5})X + \frac{1}{2}(1 \pm \sqrt{5}) = 0 \dots (VII)$ whose *four* roots are by *its origin* (by *turns*) the *eight* roots of *both* X and Y . Multiplying $(VII+) (VII-)$; is $X^4 - X^3 + 2X = 0 \dots (B)$ *two* real roots each $\dots \left\{ \begin{array}{l} X = \frac{1}{4}[1 - \sqrt{5} + \sqrt{(-2 + 6\sqrt{5})}] = Y_1 \\ X_1 = \frac{1}{4}[1 - \sqrt{5} + \sqrt{(-2 + 6\sqrt{5})}] = Y_1 \end{array} \right\} \dots (VIII)$. *Two* imaginary each $\dots \left\{ \begin{array}{l} X_2 = \frac{1}{4}[1 - \sqrt{5} + \sqrt{(-1 - 6\sqrt{5})}] = Y_a \\ X_a = \frac{1}{4}[1 + \sqrt{5} - \sqrt{(-2 - 6\sqrt{5})}] = Y_2 \end{array} \right\} = 0,809016,994375 \pm 3,926373,373101,2946\sqrt{-1} \dots (IX)$.

$\therefore \left\{ \begin{array}{l} X = -1,153721,375541,766 \text{ to } Y = 0,535687 \dots ; \\ X_1 = 0,535687,386791,872 \text{ to } Y_1 = -1,153721 \dots ; \end{array} \right.$

$$\left. \begin{array}{l} X_2 = .809016,99 \dots + 3,926373, \dots \sqrt{-1} = Y_a \\ X_a = .809016,99 \dots - 3,926373, \dots \sqrt{-1} = Y_2 \end{array} \right\} \dots (C).$$

$\therefore X + Y = X_1 + Y_1 = -0,618033,988750 -$; $X_2 + Y_2 = 1,618033,988750 = X_a + Y_a \dots (D)$. *Proof* by $XY = X + Y \dots (III_a) \left\{ \begin{array}{l} X = -1,15372137 \dots \\ Y = 0,53568738 \dots \end{array} \right\}$

Product $= -0,6180339,88 = XY$ } $\dots (XI)$. Also (C) and (A).
Sum $= -0,6180339,89 = X + Y$ }

$\left\{ \begin{array}{l} x = Ym = m(-1,1537 \dots) \text{ Product, } mXmY = xy = in \ x \\ y = Ym = m(0,5356 \dots) \text{ Sum, } mX + mY = x + y \end{array} \right.$

$$\therefore \text{ Multiplying } \left\{ \begin{array}{l} y = m_+(x+y) = m^2(-0,61803 \dots) \\ y = m_-(x+y) = m^2(-0,61802 \dots) \end{array} \right\} \dots (XII),$$

*because (III) $\dots xy = m(x+y) = m^2(-0,618033,98 \dots) = m^2XY \dots (E)$. $\therefore m$ (*vanishes*) is only a *blind* factor in x and y . \therefore by (E) ratio $x:y = X:Y$ and and (I) (II) are *fully* solved, process, ratio and roots. Q. E. D. [As, solve (III) for m . $\therefore m = \frac{1}{2}(-1 \mp \sqrt{5})(x+y) \dots$ i. e. $1 = \frac{1}{2}(-1 \mp \sqrt{5})(X+Y) \dots \therefore m$ and x and y vanish together as *always*].

COROLLARY. Two quadratics in *three* unknowns, if their every term be quadrate (in unknowns) stand for, and *solve*, the two bi-quadratics each in one *true* unknown found by treating two stated unknowns as if multiples (or other functions) of the third, and un-factoring them from it (into two *new* letters). This treatment also determines the third unknown, roots and equation, or their nature and forms when inexpressible.

COROLLARY 2. Strictly $\sqrt{m} = \pm m$. $\therefore x^2 + y^2 \pm (x+y) \dots (I_c)$ with (II_a) have *eight* roots each, viz.: (VIII, IX) or (C) direct, and *also* with signs reversed.

SCHOLIUM. General form $x^2 + y^2 + c(x+y) = a \dots (A)$; $x^2 + y^2 + xy = b \dots (B)$ solves (by quadratics) many biquadrates, decomposes surd compounds (other than of 1 with $\sqrt{5}$), finds roots, etc.